SADLER UNIT 3 MATHEMATICS SPECIALIST

WORKED SOLUTIONS

Chapter 3: Functions

Exercise 3A

Question 1

$f(x) = x+1,\ g(x) = 2x-3$, initial domain $\{0,1,2,3,4\}$

a $\{0,1,2,3,4\} \xrightarrow{f(x)} \{1,2,3,4,5\} \xrightarrow{g(x)} \{-1,1,3,5,7\}$

The range is $\{-1,1,3,5,7\}$.

b $\{0,1,2,3,4\} \xrightarrow{g(x)} \{-3,-1,1,3,5\} \xrightarrow{f(x)} \{-2,0,2,4,6\}$

The range is $\{-2,0,2,4,6\}$.

c $\{0, 1, 2, 3, 4\} \xrightarrow{g(x)} \{-3,-1, 1, 3, 5\} \xrightarrow{g(x)} \{-9,-5,-1, 3, 7\}$

The range is $\{-9,-5,-1,3,7\}$.

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Question 2

$f(x) = x + 3,\ g(x) = (x-1)^2,\ h(x) = x^3$, initial domain $\{1, 2, 3\}$

a $\{1, 2, 3\} \xrightarrow{f(x)} \{4, 5, 6\} \xrightarrow{g(x)} \{9, 16, 25\}$

The range is $\{9, 16, 25\}$.

b $\{1, 2, 3\} \xrightarrow{h(x)} \{1, 8, 27\} \xrightarrow{g(x)} \{0, 49, 676\} \xrightarrow{f(x)} \{3, 52, 679\}$

The range is $\{3, 52, 679\}$.

c $\{1, 2, 3\} \xrightarrow{f(x)} \{4, 5, 6\} \xrightarrow{g(x)} \{9, 16, 25\} \xrightarrow{h(x)} \{729, 4096, 15625\}$

The range is $\{729, 4096, 15625\}$.

Question 3

$f(x) = x + 5,\ g(x) = x - 5$

a For $f(x)$ the domain is $\{x \in \mathbb{R}\}$ and the range is $\{y \in \mathbb{R}\}$

b For $g(x)$ the domain is $\{x \in \mathbb{R}\}$ and the range is $\{y \in \mathbb{R}\}$

c For $f(x) + g(x)$ the domain is $\{x \in \mathbb{R}\}$ and the range is $\{y \in \mathbb{R}\}$

d For $f(x) - g(x)$ the domain is $\{x \in \mathbb{R}\}$ and the range is $\{y \in \mathbb{R} : y = 10\}$

e For $f(x) \bullet g(x) = x^2 - 25$ the domain is $\{x \in \mathbb{R}\}$ and the range is $\{y \in \mathbb{R} : y \geq -25\}$

f For $\dfrac{f(x)}{g(x)} = \dfrac{x+5}{x-5}$ the domain is $\{x \in \mathbb{R} : x \neq 5\}$ and the range is $\{y \in \mathbb{R} : y \neq 1\}$

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Question 4

$f(x)=3x+2,\ g(x)=\frac{2}{x},\ h(x)=\sqrt{x}$

a $\frac{2}{3x+2}=g\circ f(x)$

b $\sqrt{3x+2}=h\circ f(x)$

c $\frac{6}{x}+2=f\circ g(x)$

d $3\sqrt{x}+2=f\circ h(x)$

e $\frac{2}{\sqrt{x}}=g\circ h(x)$

f $\sqrt{\frac{2}{x}}=h\circ g(x)$

g $9x+8=f\circ f(x)$

h $x^{0\cdot25}=h\circ h(x)$

i $27x+26=f\circ f\circ f(x)$

Question 5

$f(x)=2x-3,\ g(x)=4x+1,\ h(x)=x^2+1$

a $f\circ f(x)=2(2x-3)-3=4x-9$

b $g\circ g(x)=4(4x+1)+1=16x+5$

c $h\circ h(x)=(x^2+1)^2+1=x^4+2x^2+2$

d $f\circ g(x)=2(4x+1)-3=8x-1$

e $g\circ f(x)=4(2x-3)+1=8x-11$

f $f\circ h(x)=2(x^2+1)-3=2x^2-1$

g $h\circ f(x)=(2x-3)^2+1=4x^2-12x+10$

h $g\circ h(x)=4(x^2+1)+1=4x^2+5$

i $h\circ g(x)=(4x+1)^2+1=16x^2+8x+2$

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Question 6

$f(x)=2x+5,\ g(x)=3x+1,\ h(x)=1+\frac{2}{x}$

a $f\circ f(x)=2(2x+5)+5=4x+15$

b $g\circ g(x)=3(3x+1)+1=9x+4$

c $h\circ h(x)=1+\frac{2}{1+\frac{2}{x}}=1+\frac{2}{\frac{x+2}{x}}=1+2\times\frac{x}{x+2}=\frac{x+2+2x}{x+2}=\frac{3x+2}{x+2}$

d $f\circ g(x)=2(3x+1)+5=6x+7$

e $g\circ f(x)=3(2x+5)+1=6x+16$

f $f\circ h(x)=2\left(1+\frac{2}{x}\right)+5=2+\frac{4}{x}+5=\frac{7x+4}{x}=\frac{7x}{x}+\frac{4}{x}=7+\frac{4}{x}$

g $h\circ f(x)=1+\frac{2}{2x+5}=\frac{2x+5+2}{2x+5}=\frac{2x+7}{2x+5}$

h $g\circ h(x)=3\left(1+\frac{2}{x}\right)+1=4+\frac{6}{x}$

i $h\circ g(x)=1+\frac{2}{3x+1}=\frac{3x+3}{3x+1}=\frac{3(x+1)}{3x+1}$

Question 7

$f(x)=x-4,\ g(x)=\sqrt{x}$

$g[f(x)]=\sqrt{x-4},\ x\geq 4$

Domain $\{x\in\mathbb{R}: x\geq 4\}$

Question 8

$f(x)=4-x,\ g(x)=\sqrt{x}$

$g[f(x)]=\sqrt{4-x},\ x\leq 4$

Domain $\{x\in\mathbb{R}: x\leq 4\}$

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Question 9

$f(x) = 4 - x^2,\ g(x) = \sqrt{x}$

$g[f(x)] = \sqrt{4 - x^2},\ -2 \le x \le 2$

Domain $\{x \in \mathbb{R} : -2 \le x \le 2\}$

Question 10

$g[f(x)] = \sqrt{4 - |x|}, -4 \le x \le 4$

$f(x) = 4 - |x|,\ g(x) = \sqrt{x}$

Domain $\{x \in \mathbb{R} : -4 \le x \le 4\}$

Question 11

$f(x) = x + 3,\ g(x) = \sqrt{x - 5}$

$g[f(x)] = \sqrt{x + 3 - 5} = \sqrt{x - 2},\ x \ge 2$

Domain $\{x \in \mathbb{R} : x \ge 2\}$

Question 12

$f(x) = x - 6,\ g(x) = \sqrt{x + 3}$

$g[f(x)] = \sqrt{x - 6 + 3} = \sqrt{x - 3},\ x \ge 3$

Domain $\{x \in \mathbb{R} : x \ge 3\}$

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Question 13

$f(x) = x^2 + 3,\ g(x) = \frac{1}{x}$

a $f(3) = 3^2 + 3 = 12$

b $f(-3) = (-3)^2 + 3 = 12$

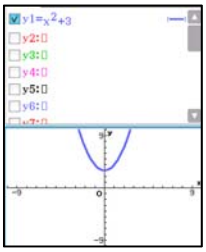
c $g(2) = \frac{1}{2}$

d $g(1) = 1$

$f(g(1)) = f(1) = 1^2 + 3 = 4$

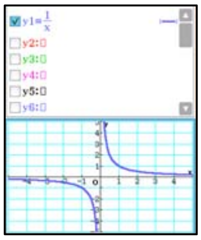
e $f(1) = 1^2 + 3 = 4$

$g(f(1)) = g(4) = \frac{1}{4}$



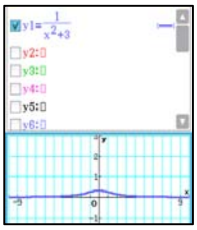
f $f(x) = x^2 + 3$ Natural domain $\{x \in \mathbb{R}\}$

Range $\{y \in \mathbb{R} : y \geq 3\}$



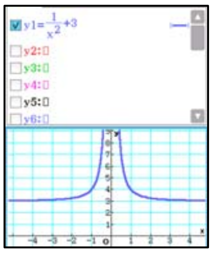
g $g(x) = \frac{1}{x}$ Natural domain $\{x \in \mathbb{R} : x \neq 0\}$

Range $\{y \in \mathbb{R} : y \neq 0\}$



h $gf(x) = \frac{1}{x^2 + 3}$ Natural domain $\{x \in \mathbb{R}\}$

Range $\left\{y \in \mathbb{R} : 0 < y \leq \frac{1}{3}\right\}$



i $fg(x) = \left(\frac{1}{x}\right)^2 + 3$ Natural domain $\{x \in \mathbb{R} : x \neq 0\}$

Range $\{y \in \mathbb{R} : y > 3\}$

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Question 14

$f(x)=25-x^2,\ g(x)=\sqrt{x}$

a $f(5)=25-5^2=0$

b $f(-5)=25-(-5)^2=0$

c $g(4)=\sqrt{4}=2$

d $fg(4)=f(\pm 2)=25-2^2=21$

e $gf(4)=g(25-16)=g(9)=\sqrt{9}=3$

f the natural domain of f is $\{x\in\mathbb{R}\}$, the range of f is $\{y\in\mathbb{R}: y\le 25\}$

g the natural domain of g is $\{x\in\mathbb{R}: x\ge 0\}$

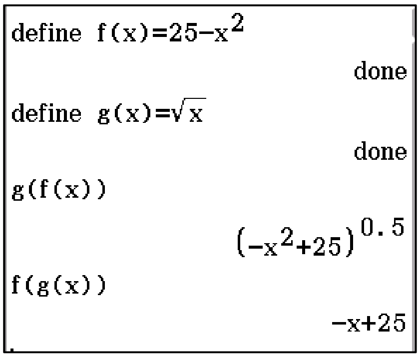
the range of g is $\{y\in\mathbb{R}: y\ge 0\}$

h the natural domain of gf is $\{x\in\mathbb{R}: -5\le x\le 5\}$

the range of gf is $\{y\in\mathbb{R}: 0\le y\le 5\}$

i the natural domain of fg is $\{x\in\mathbb{R}: x\ge 0\}$

the range of fg is $\{y\in\mathbb{R}: y\le 25\}$



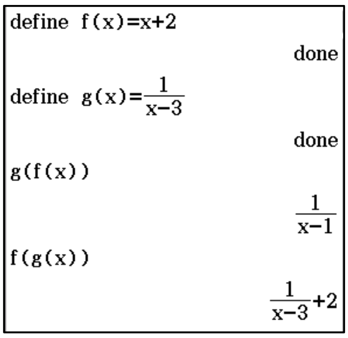
Question 15

a the natural domain of $g\circ f(x)$ is $\{x\in\mathbb{R}: x\ne 1\}$

the range of $g\circ f(x)$ is $\{y\in\mathbb{R}: y\ne 0\}$

b the natural domain of $f\circ g(x)$ is $\{x\in\mathbb{R}: x\ne 3\}$

the range of $f\circ g(x)$ is $\{y\in\mathbb{R}: y\ne 2\}$



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Question 16

a the natural domain of $g \circ f(x)$ is $\{x \in \mathbb{R} : x \geq 0\}$

the range of $g \circ f(x)$ is $\{y \in \mathbb{R} : y \geq -1\}$

b the natural domain of $f \circ g(x)$ is $\left\{x \in \mathbb{R} : x \geq \frac{1}{2}\right\}$

the range of $f \circ g(x)$ is $\{y \in \mathbb{R} : y \geq 0\}$

```
define f(x)=√x
                         done
define g(x)=2x−1
                         done
g(f(x))
                  2·x^0.5−1
f(g(x))
                 (2·x−1)^0.5
```

Question 17

a the natural domain of $g \circ f(x)$ is $\{x \in \mathbb{R} : x \neq 0\}$

the range of $g \circ f(x)$ is $\{y \in \mathbb{R} : y > 0\}$

b the natural domain of $f \circ g(x)$ is $\{x \in \mathbb{R} : x > 0\}$

the range of $f \circ g(x)$ is $\{y \in \mathbb{R} : y > 0\}$

```
define f(x)=1/x^2
                         done
define g(x)=√x
                         done
g(f(x))
                   (1/x^2)^0.5
f(g(x))
                          1/x
```

Question 18

The natural domain of $g(x)$ is $\{x \in \mathbb{R} : x > 0\}$

$f\left[g(x)\right]$ has a one-to-one relationship for $\{x \in \mathbb{R} : x > 0\}$,

and hence it is a function.

The natural domain of $f(x)$ is $\{x \in \mathbb{R}\}$,

which includes values of x which are less than zero.

And hence $g\left[f(x)\right]$ is not a function for the

natural domain of $f(x)$.

```
define f(x)=x+3
                         done
define g(x)=√x
                         done
g(f(x))
                    (x+3)^0.5
f(g(x))
                      x^0.5+3
```


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Question 19

The natural domain of $g(x)$ is $\{x \in \mathbb{R} : x \neq 5\}$

$f\left[g(x)\right]$ has a one-to-one relationship for $\{x \in \mathbb{R} : x > 0\}$,

and hence it is a function.

The natural domain of $f(x)$ is $\{x \in \mathbb{R}\}$,

which includes the value of $x = 5$ and hence $g\left[f(x)\right]$

is not a function for the natural domain of $f(x)$.

define $f(x)=x+3$
done

define $g(x)=\frac{1}{x-5}$
done

$g(f(x))$
$\frac{1}{x-2}$

$f(g(x))$
$\frac{1}{x-5}+3$

Question 20

a The natural domain of $g \circ f(x)$ is $\{x \in \mathbb{R} : x \neq \pm 3\}$

the range of $g \circ f(x)$ is $\left\{y \in \mathbb{R} : y \leq -\frac{1}{9}\right\} \cup \{y \in \mathbb{R} : y > 0\}$

b the natural domain of $f \circ g(x)$ is $\{x \in \mathbb{R} : x \neq 0\}$

the range of $f \circ g(x)$ is $\{y \in \mathbb{R} : y > -9\}$

define $f(x)=x^2-9$
done

define $g(x)=\frac{1}{x}$
done

$g(f(x))$
$\frac{1}{x^2-9}$

$f(g(x))$
$\frac{1}{x^2}-9$

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Exercise 3B

Question 1

a $f(x) = x$

Natural domain is $\{x \in \mathbb{R}\}$ and range is $\{y \in \mathbb{R}\}$.

$f^{-1}(x) = x$ with domain $\{x \in \mathbb{R}\}$ and range $\{y \in \mathbb{R}\}$.

So the function has an inverse on the natural domain.

Check: $f(3) = 3$ and $f^{-1}(3) = 3$.

b $f(x) = 2x + 3$

Natural domain is $\{x \in \mathbb{R}\}$ and range is $\{y \in \mathbb{R}\}$

$f^{-1}(x) = \dfrac{x-3}{2}$ with domain $\{x \in \mathbb{R}\}$ and range $\{y \in \mathbb{R}\}$.

So the function has an inverse on the natural domain.

Check: $f(3) = 9$ and $f^{-1}(9) = 3$.

c $f(x) = 5x - 3$

Natural domain is $\{x \in \mathbb{R}\}$ and range is $\{y \in \mathbb{R}\}$

$f^{-1}(x) = \dfrac{x+3}{5}$ with domain $\{x \in \mathbb{R}\}$ and range $\{y \in \mathbb{R}\}$.

So the function has an inverse on the natural domain.

Check: $f(3) = 12$ and $f^{-1}(12) = 3$.

d $f(x) = x^2$

Natural domain is $\{x \in \mathbb{R}\}$

The function is not one-to-one, $f(-1) = 1$ and $f(1) = 1$, so the function does not have an inverse on the natural domain.

e $f(x) = (2x-1)^2$

Natural domain is $\{x \in \mathbb{R}\}$

The function is not one-to-one, $f(0) = 1$ and $f(1) = 1$, so the function does not have an inverse on the natural domain.

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f $f(x) = x^2 + 4$

Natural domain is $\{x \in \mathbb{R}\}$

The function is not one-to-one, $f(-1) = 5$ and $f(1) = 5$, so the function does not have an inverse on the natural domain.

g $f(x) = \frac{1}{x}$

Natural domain is $\{x \in \mathbb{R} : x \neq 0\}$ and range is $\{y \in \mathbb{R} : y \neq 0\}$

$f^{-1}(x) = \frac{1}{x}$ with domain $\{x \in \mathbb{R} : x \neq 0\}$ and range $\{y \in \mathbb{R} : y \neq 0\}$, so the function has an inverse on the natural domain.

Check: $f(3) = \frac{1}{3}$ and $f^{-1}\left(\frac{1}{3}\right) = 3$.

h $f(x) = \frac{1}{x-3}$

Natural domain is $\{x \in \mathbb{R} : x \neq 3\}$ and range is $\{y \in \mathbb{R} : y \neq 0\}$

$f^{-1}(x) = \frac{1}{x} + 3$ with domain $\{x \in \mathbb{R} : x \neq 0\}$ and range $\{y \in \mathbb{R} : x \neq 3\}$

So the function has an inverse on the natural domain.

Check: $f(2) = -1$ and $f^{-1}(-1) = 2$.

i $f(x) = \frac{1}{x^2}$

Natural domain is $\{x \in \mathbb{R} : x \neq 0\}$

The function is not one-to-one, $f(-1) = 1$ and $f(1) = 1$, so the function does not have an inverse on the natural domain.

Functions **a**, **b**, **c**, **g**, **h** have inverse functions on their natural domain.

Question 2

$f(x) = x - 2$

$$x \xrightarrow{-2} x - 2$$
$$x + 2 \xleftarrow{+2} x$$

$f^{-1}(x) = x + 2$, Domain $\mathbb{R}$, Range $\mathbb{R}$.

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Question 3

$f(x) = 2x - 5$

$$x \xrightarrow{\times 2} 2x \xrightarrow{-5} 2x - 5$$
$$\frac{x+5}{2} \xleftarrow{\div 2} x + 5 \xleftarrow{+5} x$$

$f^{-1}(x) = \dfrac{x+5}{2}$, Domain $\mathbb{R}$, Range $\mathbb{R}$.

Question 4

$f(x) = 5x + 2$

$$x \xrightarrow{\times 5} 5x \xrightarrow{+2} 5x + 2$$
$$\frac{x-2}{5} \xleftarrow{\div 5} x - 2 \xleftarrow{-2} x$$

$f^{-1}(x) = \dfrac{x-2}{5}$, Domain $\mathbb{R}$, Range $\mathbb{R}$.

Question 5

$f(x) = \dfrac{1}{x-4}$

$$x \xrightarrow{-4} x - 4 \xrightarrow{Invert} \frac{1}{x-4}$$
$$\frac{1}{x} + 4 \xleftarrow{+4} \frac{1}{x} \xleftarrow{Invert} x$$

$f^{-1}(x) = \dfrac{1}{x} + 4$, Domain $x \neq 0$, Range $y \neq 4$.

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Question 6

$$f(x)=\frac{1}{x+3}$$

$$\begin{array}{l} x \xrightarrow{+3} x+3 \xrightarrow{Invert} \dfrac{1}{x+3} \\ \dfrac{1}{x}-3 \xleftarrow{-3} \dfrac{1}{x} \xleftarrow{Invert} x \end{array}$$

$f^{-1}(x)=\dfrac{1}{x}-3$, Domain $x\neq 0$, Range $y\neq -3$.

Question 7

$$f(x)=\frac{1}{2x-5}$$

$$\begin{array}{l} x \xrightarrow{\times 2} 2x \xrightarrow{-5} 2x-5 \xrightarrow{Invert} \dfrac{1}{2x-5} \\ \dfrac{1}{2}\left(\dfrac{1}{x}+5\right) \xleftarrow{\div 2} \dfrac{1}{x}+5 \xleftarrow{+5} \dfrac{1}{x} \xleftarrow{Invert} x \end{array}$$

$f^{-1}(x)=\dfrac{1}{2}\left(\dfrac{1}{x}+5\right)=\dfrac{1}{2x}+\dfrac{5}{2}=\dfrac{1+5x}{2x}$, Domain $x\neq 0$, Range $y\neq 2.5$.

Question 8

$$f(x)=1+\frac{1}{2+x}$$

$$\begin{array}{l} x \xrightarrow{+2} x+2 \xrightarrow{Invert} \dfrac{1}{x+2} \xrightarrow{+1} 1+\dfrac{1}{x+2} \\ \dfrac{1}{x-1}-2 \xleftarrow{-2} \dfrac{1}{x-1} \xleftarrow{Invert} x-1 \xleftarrow{-1} x \end{array}$$

$f^{-1}(x)=\dfrac{1}{x-1}-2$, Domain $x\neq 1$, Range $y\neq -2$.

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Question 9

$f(x) = 3 - \frac{1}{x-1}$

$$x \xrightarrow{-1} x-1 \xrightarrow{Invert} \frac{1}{x-1} \xrightarrow{\times(-1)} -\frac{1}{x-1} \xrightarrow{+3} 3-\frac{1}{x-1}$$

$$\frac{1}{3-x}+1 \xleftarrow{+1} \frac{1}{3-x} \xleftarrow{Invert} 3-x \xleftarrow{\div(-1)} x-3 \xleftarrow{-3} x$$

$f^{-1}(x) = 1 - \frac{1}{x-3} = \frac{1}{3-x} + 1$, Domain $x \neq 3$, Range $y \neq 1$.

Question 10

$f(x) = 4 + \frac{2}{2x-1}$

$$x \xrightarrow{\times 2} 2x \xrightarrow{-1} 2x-1 \xrightarrow{Invert} \frac{1}{2x-1} \xrightarrow{\times 2} \frac{2}{2x-1} \xrightarrow{+4} 4+\frac{2}{2x-1}$$

$$\frac{1}{x-4}+\frac{1}{2} \xleftarrow{\div 2} \frac{2}{x-4}+1 \xleftarrow{+1} \frac{2}{x-4} \xleftarrow{Invert} \frac{x-4}{2} \xleftarrow{\div 2} x-4 \xleftarrow{-4} x$$

$f^{-1}(x) = \frac{1}{x-4} + \frac{1}{2}$, Domain $x \neq 4$, Range $y \neq 0.5$.

Question 11

$f(x) = \sqrt{x}$

$$x \xrightarrow{\sqrt{x}} \sqrt{x}$$

$$x^2 \xleftarrow{x^2} x$$

$f^{-1}(x) = x^2$, Domain $x \geq 0$, Range $y \geq 0$.

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Question 12

$f(x) = \sqrt{x+1}$

$$\begin{array}{l} x \xrightarrow{+1} x+1 \xrightarrow{\sqrt{x}} \sqrt{x+1} \\ x^2 - 1 \xleftarrow{-1} x^2 \xleftarrow{x^2} x \end{array}$$

$f^{-1}(x) = x^2 - 1$, Domain $x \geq 0$, Range $y \geq -1$.

Question 13

$f(x) = \sqrt{2x-3}$

$$\begin{array}{l} x \xrightarrow{\times 2} 2x \xrightarrow{-3} 2x-3 \xrightarrow{\sqrt{x}} \sqrt{2x-3} \\ \dfrac{x^2+3}{2} \xleftarrow{\div 2} x^2+3 \xleftarrow{+3} x^2 \xleftarrow{x^2} x \end{array}$$

$f^{-1}(x) = \dfrac{x^2+3}{2}$, Domain $x \geq 0$, Range $y \geq 1.5$.

Question 14

$f(x) = 2x + 5$

$$\begin{array}{l} x \xrightarrow{\times 2} 2x \xrightarrow{+5} 2x+5 \\ \dfrac{x-5}{2} \xleftarrow{\div 2} x-5 \xleftarrow{-5} x \end{array}$$

$f^{-1}(x) = \dfrac{x-5}{2}$

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Question 15

$g(x) = 3x + 1$

$$x \xrightarrow{\times 3} 3x \xrightarrow{+1} 3x + 1$$
$$\frac{x-1}{3} \xleftarrow{\div 3} x - 1 \xleftarrow{-1} x$$

$g^{-1}(x) = \dfrac{x-1}{3}$

Question 16

$h(x) = 1 + \dfrac{2}{x}$

$$x \xrightarrow{Invert} \frac{1}{x} \xrightarrow{\times 2} \frac{2}{x} \xrightarrow{+1} 1 + \frac{2}{x}$$
$$\frac{2}{x-1} \xleftarrow{Invert} \frac{x-1}{2} \xleftarrow{\div 2} x - 1 \xleftarrow{-1} x$$

$h^{-1}(x) = \dfrac{2}{x-1}$

Question 17

$f \circ f^{-1}(x) = 2\left(\dfrac{x-5}{2}\right) + 5 = x$

Question 18

$f^{-1} \circ f(x) = \dfrac{2x + 5 - 5}{2} = x$

Question 19

$f \circ h^{-1}(x) = 2\left(\dfrac{2}{x-1}\right) + 5 = \dfrac{4}{x-1} + 5$

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Question 20

$$f \circ g(x) = 2(3x+1)+5 = 6x+7$$

$$\left(f \circ g\right)^{-1}(x) = \frac{x-7}{6}$$

Question 21

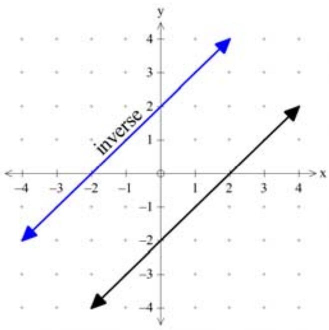
$$g^{-1} \circ f^{-1}(x) = \frac{\frac{x-5}{2}-1}{3} = \frac{1}{3}\left(\frac{x-7}{2}\right) = \frac{x-7}{6}$$

Question 22

$$f \circ g^{-1}(x) = 2\left(\frac{x-1}{3}\right)+5 = \frac{2x+13}{3}$$

Question 23

a Is a function, passes the vertical line test, one-to-one.



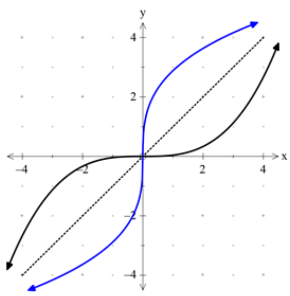
b Is not a function, for some values of x there is more than one y value.

c Is a function, not one-to-one.

d Is not function, for some values of x there is more than one y value.

e Is not function, for some values of x there is more than one y value.

f Is a function, passes the vertical line test, one-to-one.



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Question 24

For $f(x)$ restricted to $x \ge 0$ then $f^{-1}(x) = \sqrt{x-3}$, domain $x \ge 3$ and range $y \ge 0$.

(or $f(x)$ restricted to $x \le 0$ then $f^{-1}(x) = -\sqrt{x-3}$, domain $x \ge 3$ and range $y \le 0$.)

Question 25

For $f(x)$ restricted to $x \ge -3$ then $f^{-1}(x) = -3 + \sqrt{x}$, domain $x \ge 0$ and range $y \ge -3$.

(or $f(x)$ restricted to $x \le -3$ then $f^{-1}(x) = -3 - \sqrt{x}$, domain $x \ge 0$ and range $y \le -3$.)

Question 26

For $f(x)$ restricted to $x \ge 3$ then $f^{-1}(x) = 3 + \sqrt{x-2}$, domain $x \ge 2$ and range $y \ge 3$.

(or $f(x)$ restricted to $x \le 3$ then $f^{-1}(x) = 3 - \sqrt{x-2}$, domain $x \ge 2$ and range $y \le 3$.)

Question 27

For $f(x)$ restricted to $0 \le x \le 2$ then $f^{-1}(x) = \sqrt{4-x^2}$, domain $0 \le x \le 2$ and range $0 \le y \le 2$.

(or $f(x)$ restricted to $-2 \le x \le 0$ then $f^{-1}(x) = -\sqrt{4-x^2}$, domain $0 \le x \le 2$ and range $-2 \le y \le 0$.)

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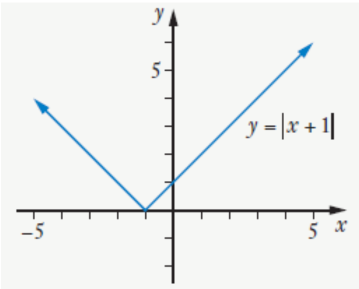
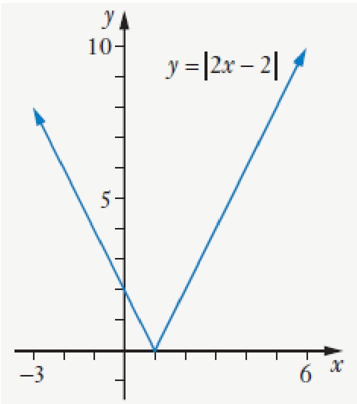
Exercise 3C

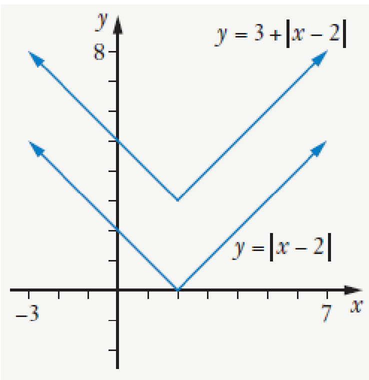
Question 1



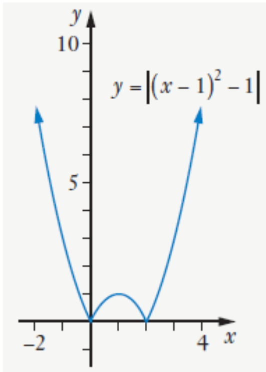
Question 2



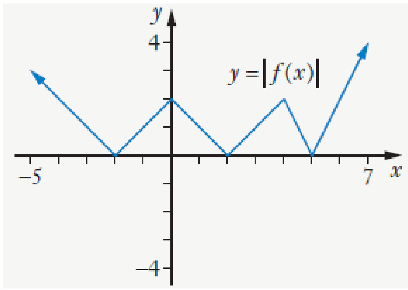
Question 3



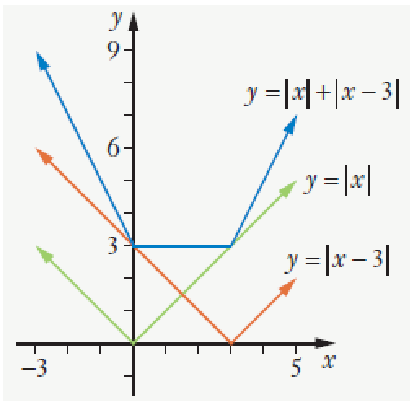
Question 4



Question 5



Question 6



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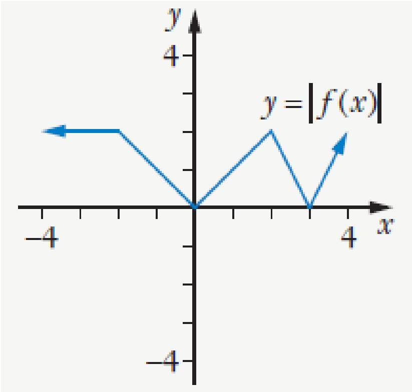
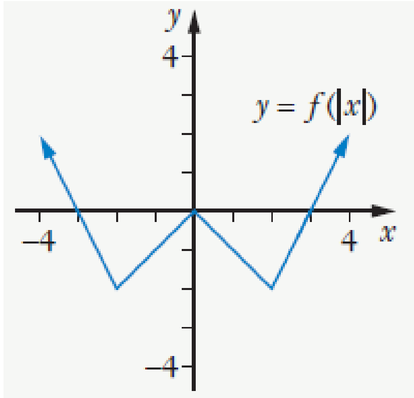
Question 7

a

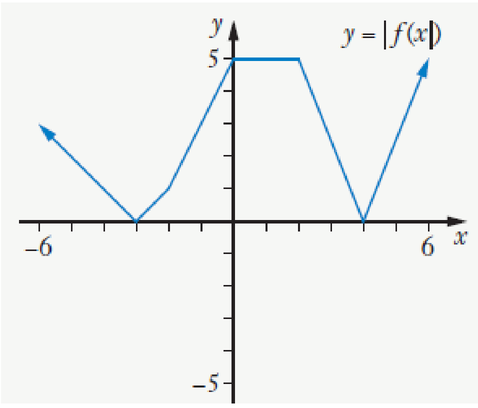


b

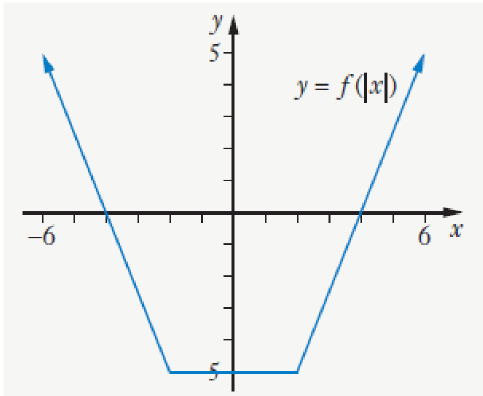


Question 8

a

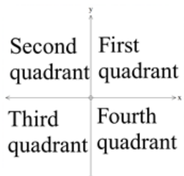


b



Question 9

In the first and fourth quadrants (see the diagram) the graph of $y = g(|x|)$ will be the same as that of $y = g(x)$. However, in the second and third quadrants the graph of $y = g(|x|)$ will be those parts of $y = g(x)$ that lie in the first and fourth quadrants, reflected about the y-axis.



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Question 10

a The function $g(x)=(x+1)^2$ has domain $\mathbb{R}$ and range $\{y\in\mathbb{R}: y\geq 0\}$.

The function $f(x)=2+\sqrt{x}$ has domain $\{x\in\mathbb{R}: x\geq 0\}$ and range $\{y\in\mathbb{R}: y\geq 2\}$.

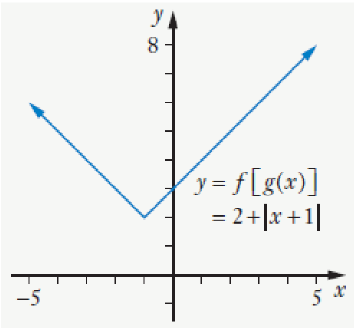
Thus $g(x)$ is defined for all real x and the output from $g(x)$ consists of numbers that are all within the domain of $f(x)$. Thus $f[g(x)]$ is defined for all real x.

$$\mathbb{R}\rightarrow \boxed{g(x)=(x+1)^2}\rightarrow y\in\mathbb{R}: y\geq 0\rightarrow \boxed{f(x)=2+\sqrt{x}}\rightarrow y\in\mathbb{R}: y\geq 2$$

Thus $f[g(x)]$ has domain $\mathbb{R}$ and range $\{y\in\mathbb{R}: y\geq 2\}$.

b $f[g(x)]=2+\sqrt{(x+1)^2}=2+|x+1|$

c



Question 11

a The function $g(x)=(x-2)^2$ has domain $\mathbb{R}$ and range $\{y\in\mathbb{R}: y\geq 0\}$.

The function $f(x)=1-\sqrt{x}$ has domain $\{x\in\mathbb{R}: x\geq 0\}$ and range $\{y\in\mathbb{R}: y\leq 1\}$.

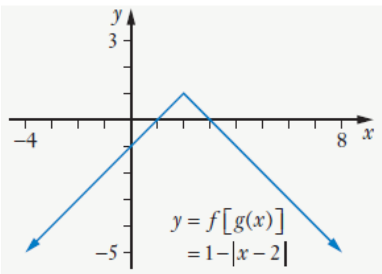
Thus $g(x)$ is defined for all real x and the output from $g(x)$ consists of numbers that are all within the domain of $f(x)$. Thus $f[g(x)]$ is defined for all real x.

$$\mathbb{R}\rightarrow \boxed{g(x)=(x-2)^2}\rightarrow y\in\mathbb{R}: y\geq 0\rightarrow \boxed{f(x)=1-\sqrt{x}}\rightarrow y\in\mathbb{R}: y\leq 1$$

Thus $f[g(x)]$ has domain $\mathbb{R}$ and range $\{y\in\mathbb{R}: y\leq 1\}$.

b $f[g(x)]=1-\sqrt{(x-2)^2}=1-|x-2|$

c



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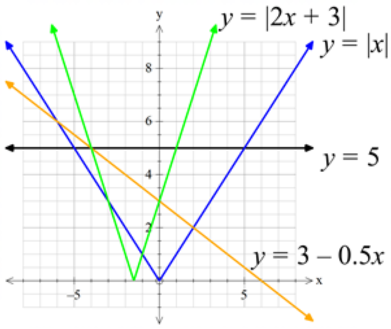
Question 12

a $x = 3, 7$

b $x = -2,\ 6$

c $x = 4,\ 8$

Question 13



a $x = 1,\ -4$

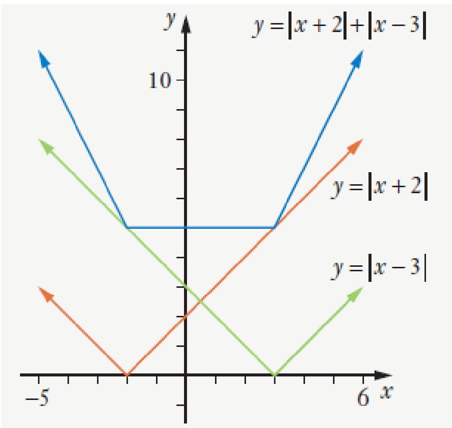
b $x = -6,\ 2$

c $x = 0,\ -4$

d $x = -1,\ -3$

Question 14

a, **b** and **c**



d $|x+2| + |x-3| \le 9$ for $-4 \le x \le 5$.

Question 15

$$|x+6| = 1$$

$$\begin{aligned} x + 6 &= 1 \qquad & -x - 6 &= 1 \\ x &= -5 & -x &= 7 \\ & & x &= -7 \end{aligned}$$

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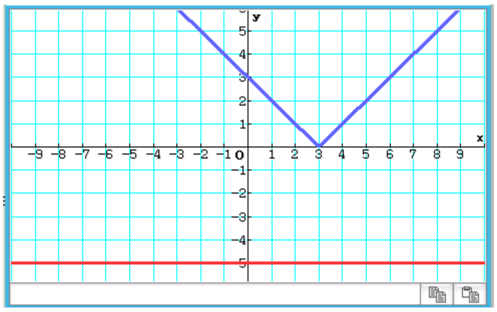
Question 16

$|x-3|=-5$

There is no solution as the magnitude of any number must be positive and not negative.



Question 17

$|x-10|=|x-6|$

$x-10=-x+6 \qquad -x+10=-x+6$

$2x=16 \qquad 10\neq 16$

$x=8$ is the only solution

Question 18

$|x+5|=|2x-14|$

$x+5=2x-14 \qquad -x-5=2x-14$

$x=19 \qquad 3x=9$

$x=3$

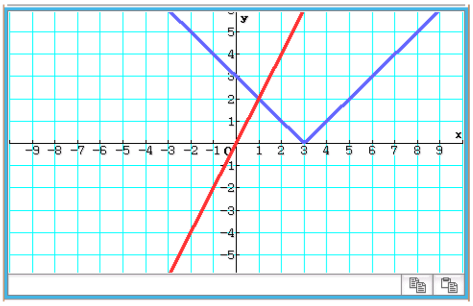
Question 19

$|x-3|=2x$

$-x+3=2x$

$3x=3$

$x=1$



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Question 20

$|x+5|+|x-1|=7$

$x+5+x-1=7 \qquad -x-5-x+1=7$

$2x=3 \qquad -2x=11$

$x=1.5 \qquad x=-5.5$

Question 21

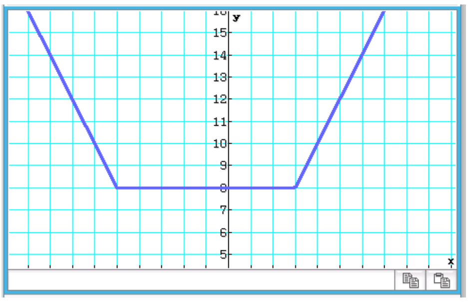
$|x+5|+|x-3|=8$

$x+5+x-3=8 \qquad -x-5-x+3=8$

$2x=6 \qquad -2x=10$

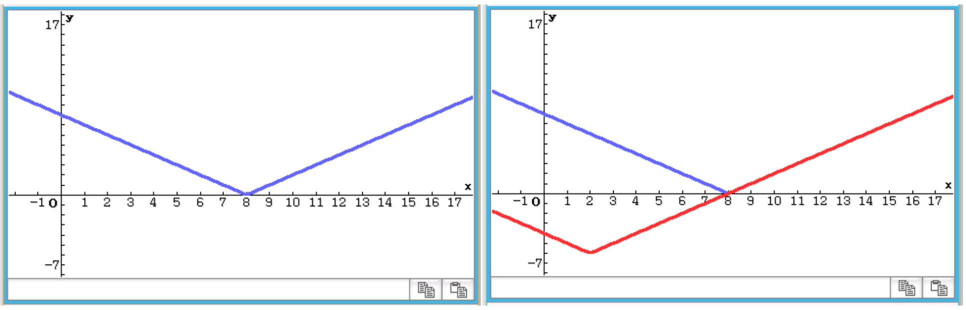
$x=3 \qquad x=-5$

$-5 \le x \le 3$



Question 22

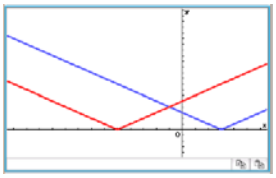
$x \ge 8$



Question 23

$|x-3| \ge |x+5|$

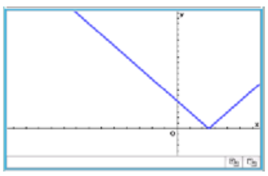
$x \le -1$



Question 24

$|2x-5| \ge -5$

$x \in \mathbb{R}$



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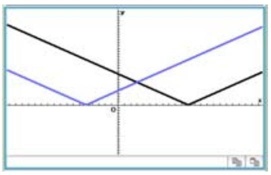
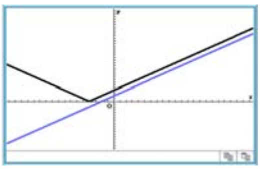
Question 25

$$|x-11| \geq |x+5|$$
$$x \leq 3$$



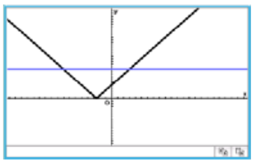
Question 26

$$|x+4| > x+2$$
$$x \in \mathbb{R}$$



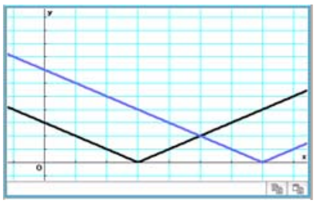
Question 27

$$|2x+5| > a$$
$$2(3)+5 = a$$
$$11 = a$$
$$-2b-5 = 11$$
$$b = -8$$



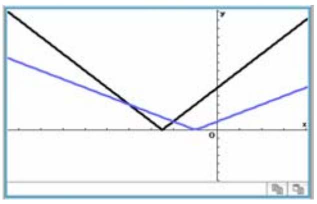
Question 28

$$|x-3| \leq |x-a|$$
$$-5+3 = 5-a$$
$$7 = a$$
$$|x-3| \leq |x-7|$$



Question 29

$$|2x+5| = x+a$$
$$2(-4)+5 = -4+a$$
$$a = 1$$
$$|2x+5| < x+1$$



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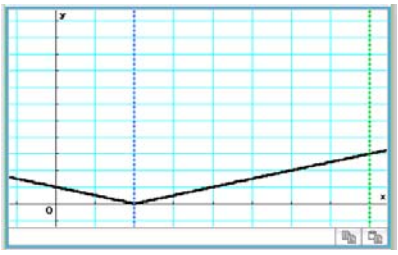
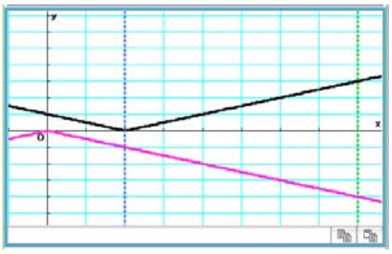
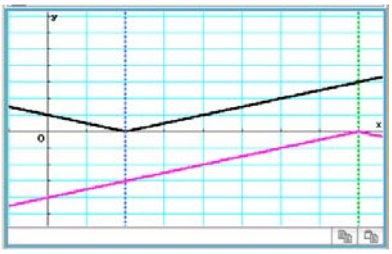
Question 30



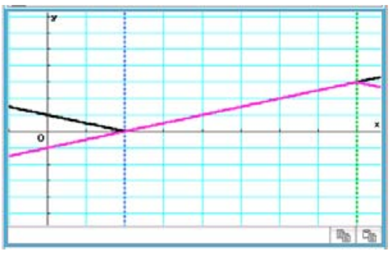
$a = -0.5$



$b = 8$



$c = 3$



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Exercise 3D

Question 1

$y = \frac{2}{x}$

Vertical asymptote at $x = 0$.

Question 2

$y = \frac{5}{x-1}$

Vertical asymptote at $x = 1$.

Question 3

$y = \frac{5}{(x-3)(2x-1)}$

Vertical asymptotes at $x = 3, x = \frac{1}{2}$.

Question 4

$y = \frac{x+3}{x-3}$

Vertical asymptotes at $x = 3$.

Question 5

$y = \frac{3}{x}$

$y = 0$ cannot be obtained.

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Question 6

$$y = 2 + \frac{3}{x}$$

$y = 2$ cannot be obtained.

Question 7

$$y = \frac{1}{x+1}$$

$y = 0$ cannot be obtained.

Question 8

$$y = \frac{x-1}{x+1}$$

$y = 1$ cannot be obtained.

Question 9

$$y = \frac{1}{x-5}$$

As $x \to +\infty$ then $y \to 0^+$

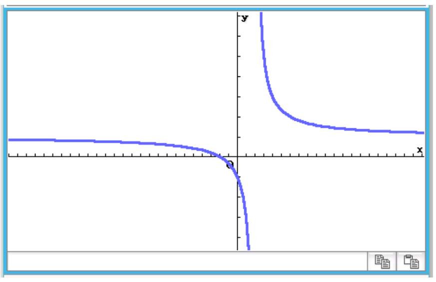
As $x \to -\infty$ then $y \to 0^-$

Question 10

$$y = \frac{x+2}{x-2}$$

As $x \to +\infty$ then $y \to 1^+$

As $x \to -\infty$ then $y \to 1^-$



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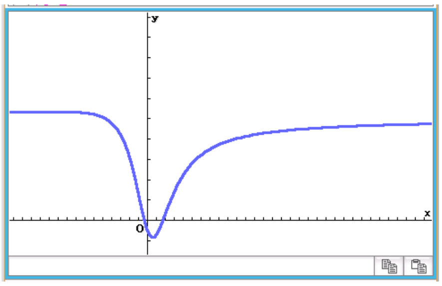
Question 11

$$y = \frac{5x^2 + 7x - 3}{x^2 + 6}$$



As $x \to +\infty$ then $y \to 5^+$

As $x \to -\infty$ then $y \to 5^-$

Question 12

$$y = \frac{3x(x+2)}{x^2 + 1}$$

As $x \to +\infty$ then $y \to 3^+$

As $x \to -\infty$ then $y \to 3^-$

Question 13

$$y = \frac{1}{x - 3}$$

As $x \to 3^+$ then $y \to +\infty$

As $x \to 3^-$ then $y \to -\infty$

Question 14

$$y = \frac{1}{1 - x}$$

As $x \to 1^+$ then $y \to -\infty$

As $x \to 1^-$ then $y \to +\infty$

Question 15

$$y = \frac{x^5 + 1}{x^2}$$

As $x \to 0^+$ then $y \to +\infty$

As $x \to 0^-$ then $y \to +\infty$

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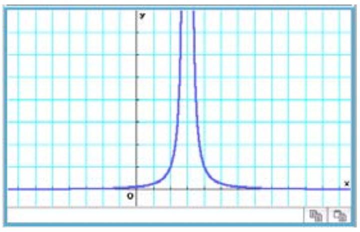
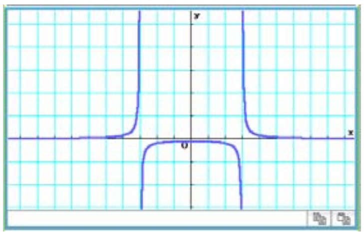
Question 16

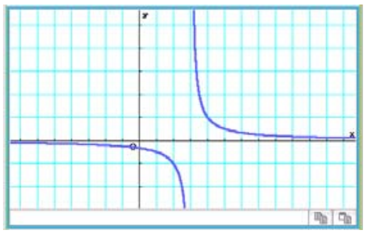
a $y = \dfrac{1}{(x-3)^2}$



b $y = \dfrac{1}{(x+3)(x-3)}$



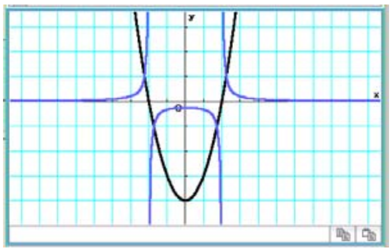
c $y = \dfrac{1}{x-3}$



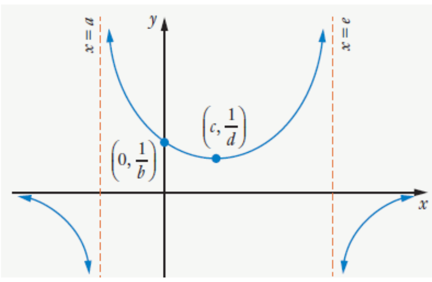
Question 17

y1=(x+2)•(x−2)

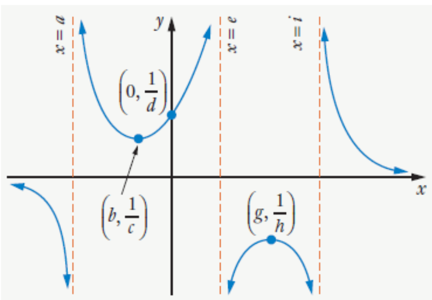
$y2=\dfrac{1}{(x+2)\cdot(x-2)}$



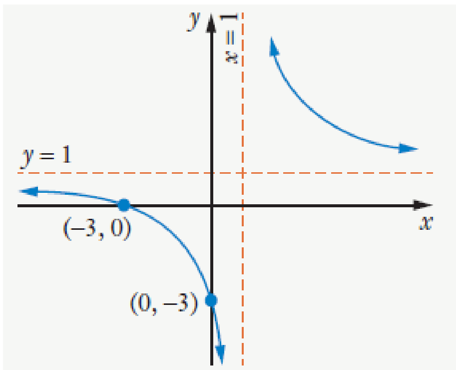
Question 18



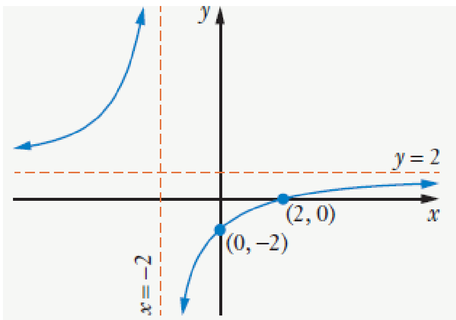
Question 19



Question 20



Question 21



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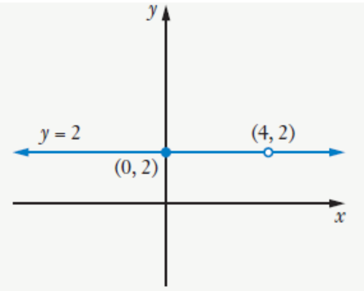
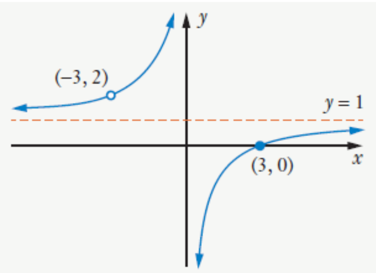
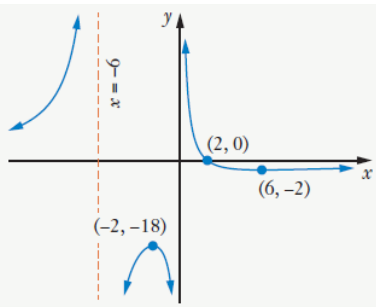
Question 22



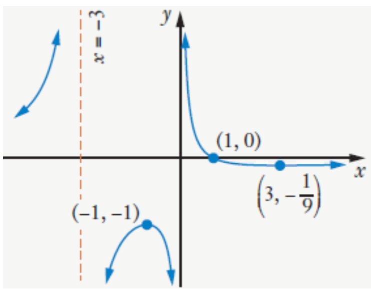
Question 23



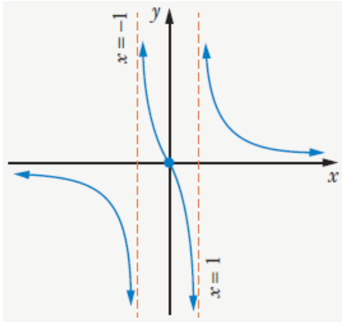
Question 24



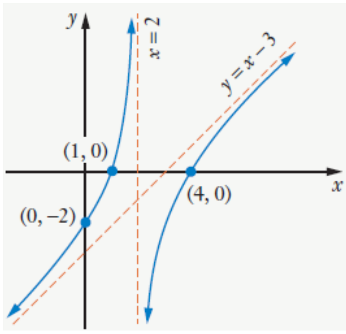
Question 25



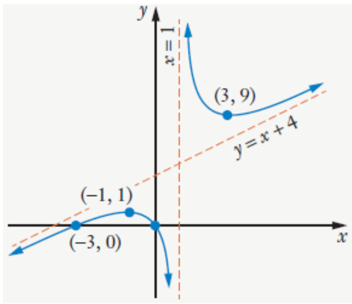
Question 26



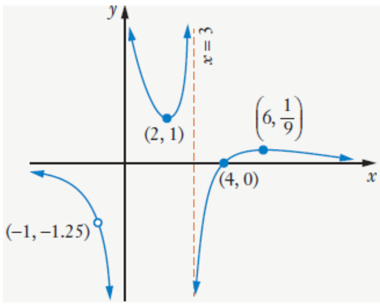
Question 27



Question 28



Question 29



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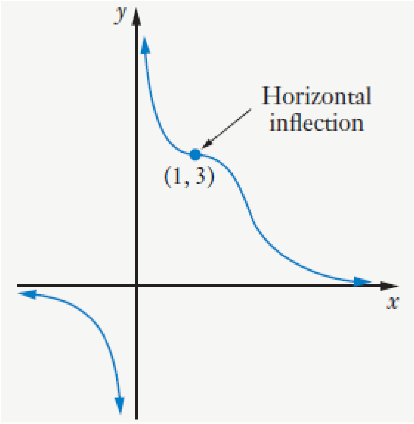
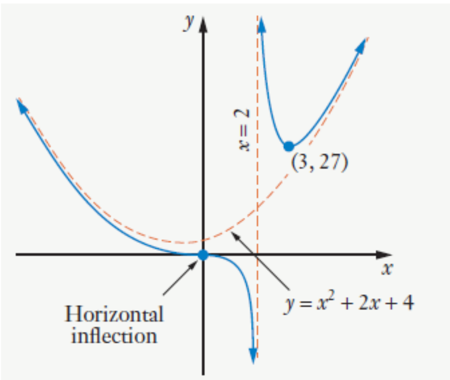
Question 30



Question 31



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Miscellaneous Exercise 3

Question 1

$$x^3 + 7x^2 + 19x + 13 = 0$$

$$(x+1)(x^2 + 6x + 13) = 0$$

$$(x+1)\left(x+3+\frac{1}{2}\sqrt{-16}\right)\left(x+3-\frac{1}{2}\sqrt{-16}\right) = 0$$

$$x = -1, -3+2i, -3-2i$$

Question 2

a Asymptotes occur when $x-1=0$ and $x+3=0$.

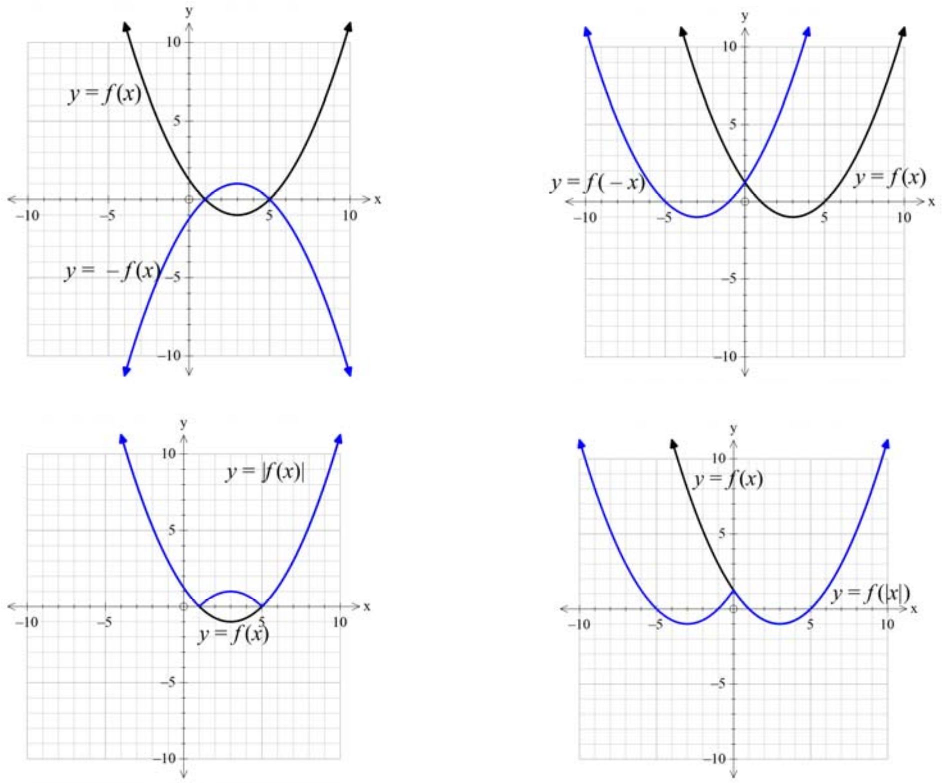
$a = -3$ and $b = 1$.

b $y = (x-1)(x+3) = x^2 + 2x - 3 = (x+1)^2 - 4$

When $x = -1$, $y = \dfrac{1}{(-1-1)(-1+3)} = -\dfrac{1}{4}$

Point C has coordinates $\left(-1, -\dfrac{1}{4}\right)$

Question 3



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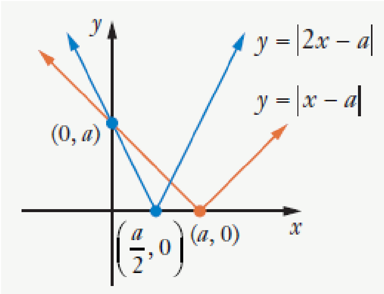
Question 4



$0 \le x \le \frac{2}{3}a$

Question 5

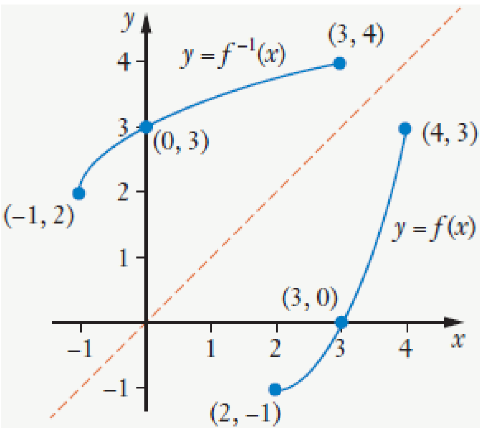
a Domain $\{x \in \mathbb{R} : 2 \le x \le 4\}$

Range $\{y \in \mathbb{R} : -1 \le y \le 3\}$

b Domain $\{x \in \mathbb{R} : -1 \le x \le 3\}$

Range $\{y \in \mathbb{R} : 2 \le y \le 4\}$

c



d $f^{-1}(x) = \sqrt{x+1} + 2$

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Question 6

a $p\mathbf{a} = q\mathbf{b}$, in order for this to be true $p = q = 0$

b $(p-3)\mathbf{a} = q\mathbf{b}$, in order for this to be true $p = 3$ and $q = 0$.

c $(p+2)\mathbf{a} = (q-1)\mathbf{b}$, in order for this to be true $p = -2$ and $q = 1$.

d $p\mathbf{a} + 2\mathbf{b} = 3\mathbf{a} - q\mathbf{b}$, which is only true if $p = 3$ and $q = -2$.

e $p\mathbf{a} + q\mathbf{a} + p\mathbf{b} - 2q\mathbf{b} = 3\mathbf{a} + 6\mathbf{b}$

$$\begin{cases} p+q=3 \\ p-2q=6 \end{cases}$$

$3q = -3 \quad \Rightarrow \quad q = -1, p = 4$

f $p\mathbf{a} + 2\mathbf{a} - 2p\mathbf{b} = \mathbf{b} + 5q\mathbf{b} - q\mathbf{a}$

p$\mathbf{a} + 2\mathbf{a} - 2p\mathbf{b} = \mathbf{b} + 5q\mathbf{b} - q\mathbf{a}$

Equating coefficients:

$p + 2 = -q$ [1]

$-2p = 1 + 5q$ [2]

Substituting [1] into [2]:

$-2p = 1 + 5(-p-2)$

$3p = -9 \quad \Rightarrow \quad p = -3, q = 1$

Question 7

$\mathbf{a} = -9\mathbf{i} + 21\mathbf{j}$ and $\mathbf{b} = 5\mathbf{i} - 3\mathbf{j}$

a

$2p + 5q = -9$

$4p - 3q = 21$

$13q = -39 \quad \Rightarrow \quad q = -3, p = 3$

$-9\mathbf{i} + 21\mathbf{j} = 3\mathbf{a} - 3\mathbf{b}$

b

$2p + 5q = 4$

$4p - 3q = -18$

$13q = 26 \quad \Rightarrow \quad q = 2, p = -3$

$4\mathbf{i} - 18\mathbf{j} = -3\mathbf{a} + 2\mathbf{b}$

c

$2p + 5q = -7$

$4p - 3q = 12$

$13q = -26 \quad \Rightarrow \quad q = -2, p = 1.5$

$-7\mathbf{i} + 12\mathbf{j} = \frac{3}{2}\mathbf{a} - 2\mathbf{b}$

d

$2p + 5q = -34$

$4p - 3q = 23$

$13q = -91 \quad \Rightarrow \quad q = -7, p = 0.5$

$-34\mathbf{i} + 23\mathbf{j} = \frac{1}{2}\mathbf{a} - 7\mathbf{b}$

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Question 8

a

$$z=\frac{3+5\sqrt{3}i}{-3+2\sqrt{3}i}\times\frac{-3-2\sqrt{3}i}{-3-2\sqrt{3}i}$$
$$=\frac{-9-6\sqrt{3}i-15\sqrt{3}i-30i^2}{9-12i^2}$$
$$=\frac{21-21\sqrt{3}i}{21}=1-\sqrt{3}i$$

b

$$r=\sqrt{1^2+(-\sqrt{3})^2}=2$$
$$\tan\theta=-\sqrt{3}$$
$$\theta=-\frac{\pi}{3}$$
$$z=2\text{cis}\left(-\frac{\pi}{3}\right)$$

Question 9

$$\frac{1}{4\text{cis}\left(-\frac{\pi}{6}\right)}=\frac{1}{4}\text{cis}\left(\frac{\pi}{6}\right)$$

$$a=\frac{1}{4}\cos\frac{\pi}{6}=\frac{\sqrt{3}}{8},\qquad b=\frac{1}{4}\sin\frac{\pi}{6}=\frac{1}{8}$$
$$z=\frac{\sqrt{3}}{8}+i\frac{1}{8}$$

Question 10

$p=iz,\quad q=-z,\quad w=-iz$

Question 11

$z=2\,\text{cis}\frac{\pi}{4}$ and $w=1\,\text{cis}\frac{\pi}{6}$

a

$$zw=2\text{cis}\left(\frac{\pi}{4}+\frac{\pi}{6}\right)=2\text{cis}\frac{5\pi}{12}$$

b

$$\frac{z}{w}=2\text{cis}\left(\frac{\pi}{4}-\frac{\pi}{6}\right)=2\text{cis}\left(\frac{\pi}{12}\right)$$

c

$$w^2=1\,\text{cis}\left(\frac{\pi}{6}+\frac{\pi}{6}\right)=1\,\text{cis}\frac{\pi}{3}$$

d

$$z^3=8\text{cis}\frac{3\pi}{4}$$

e

$$w^9=1\,\text{cis}\frac{9\pi}{6}$$
$$=1\,\text{cis}\frac{3\pi}{2}\ \text{(not in the domain)}$$
$$=1\text{cis}\left(-\frac{\pi}{2}\right)$$

f

$$z^9=512\text{cis}\frac{9\pi}{4}\ \text{(not in the domain)}$$
$$=512\text{cis}\frac{\pi}{4}$$

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Question 12

$$-\sqrt{3}+i = 2\,\text{cis}\,\frac{5\pi}{6}$$

$$\begin{aligned}\left(-\sqrt{3}+i\right)^{12} &= 2^{12}\,\text{cis}\left(12\times\frac{5\pi}{6}\right) \text{ (by de Moivre's theorem)}\\ &= 4096\left[\cos(0)+i\sin(0)\right]\\ &= 4096\end{aligned}$$

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